Exercise 60

At what point on the curve $y = \sqrt{1 + 2x}$ is the tangent line perpendicular to the line $6x + 2y = 1$.

Solution

Solve the given equation of the line for y.

$$y = -3x + \frac{1}{2}$$

The slope of the line perpendicular to this one is $1/3$. Take the derivative of $y = \sqrt{1 + 2x}$ and determine the value of x where it's $1/3$.

$$y' = \frac{dy}{dx} = \frac{d}{dx}\sqrt{1 + 2x}$$

$$= \frac{1}{2}(1 + 2x)^{-1/2} \cdot \frac{d}{dx}(1 + 2x)$$

$$= \frac{1}{2}(1 + 2x)^{-1/2} \cdot (2)$$

$$= \frac{1}{\sqrt{1 + 2x}}$$

Set it equal to $1/3$ and solve for x.

$$\frac{1}{\sqrt{1 + 2x}} = \frac{1}{3} \quad \rightarrow \quad x = 4$$

Plug this value of x into the function for y.

$$y(4) = \sqrt{1 + 2(4)} = 3$$

Therefore, the point on the curve $y = \sqrt{1 + 2x}$ for which the tangent line is perpendicular to the line $6x + 2y = 1$ is

$$(4, 3).$$